NETWORK

LAPLACE TRANSFORM

Laplace transform:- If $f(t)$ is a function defined for $t>0$, then the laplace transform of function $f(t)$ is denoted by $L\{f(t)\}$ and given by

$$\boxed{L\{f(t)\} = \int_0^\infty f(t).e^{-st}\,dt}$$

$$L\{f(t)\} = F(s) = \int_0^\infty f(t).e^{-st}\,dt$$

where

s = complex frequency variable and equal $\sigma + j\omega$.

$\sigma \rightarrow$ is the real part and

$\omega \rightarrow$ is the imaginary quantity and σ, ω are real numbers

Laplace transform of some standard functions.

1) Exponential function:-

$$f(t) = \begin{cases} 0 \ ; & t<0 \\ ke^{-at} ; & t\geq 0 \end{cases}$$

where

k and a are constants.

The Laplace transform of $f(t)$ is

$$L\{f(t)\} = F(s) = \int_0^\infty k.e^{-at}.e^{-st}\,dt$$

$$= \int_0^\infty k.e^{-(a+s)t}\,dt$$

$$= k.\left[\frac{e^{-(a+s)}}{-(a+s)}\right]_0^\infty$$

$$= \frac{-k}{(a+s)}\left[0-1\right] = \frac{k}{s+a} \text{ Ans}$$

II) Step function:

Step funct is defined as

$$f(t) = u(t) = \begin{cases} 0 & ; t < 0 \\ A & ; t > 0 \end{cases}$$

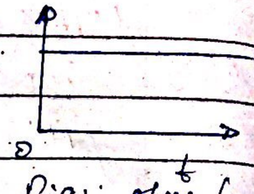

Fig: step function

If $A = 1$ then it is called unit step function and denoted by $u(t)$.

$$L\{f(t)\} = F(s) = \int_0^\infty A e^{-st} dt = \frac{A\left[e^{-st}\right]_0^\infty}{-s}$$

$$= \frac{-A}{s}[0 - 1]$$

$$= \frac{A}{s} = \frac{1}{s} \quad [\text{if } A = 1]$$

III) Ramp function:

The ramp function is denoted by $r(t)$ and defined as

$$f(t) = \begin{cases} 0 & ; t < 0 \\ At & ; t \geq 0 \end{cases}$$

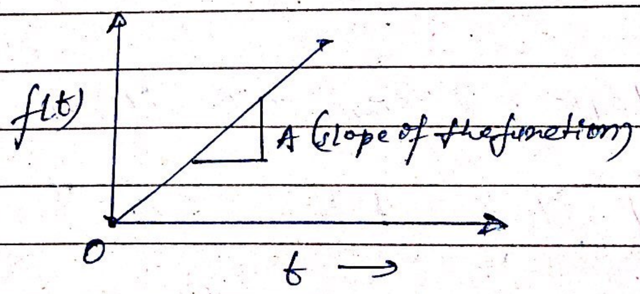

Fig: Ramp function

Now, take the laplace transform we get

$$L\{f(t)\} = F(s) = \int_0^\infty At\, e^{-st} dt$$

using ILATE we get

$$F(s) = A\left\{\left[\frac{t e^{-st}}{-s}\right]_0^\infty - \left[\frac{1 \cdot e^{-st}}{s^2}\right]_0^\infty\right\}$$

$$= A\left\{\frac{\infty - 0}{-s} - \frac{1}{s^2}(0-1)\right\}$$

$$= A\{\infty\} + \frac{1}{s^2}A$$

$$\therefore L\{f(t)\} = \frac{A}{s^2} \quad \text{Ans}$$

IV.) Sinusodial Function:

Sinusodial function is defined as

$$f(t) = \begin{cases} 0 \,; & t < 0 \\ A\sin\omega t \,; & t \geq 0 \end{cases}$$

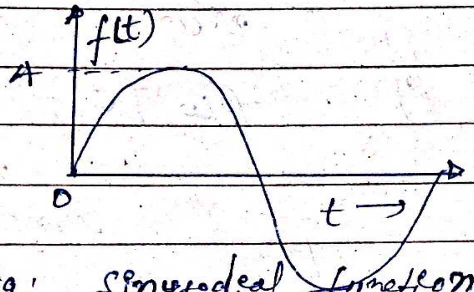

Fig: Sinusodial function.

Now take laplace transform we gets.

$$L\{f(t)\} = F(s) = \int_0^\infty A\sin\omega t \, dt \, e^{-st}$$

$$= \int_0^\infty A\sin\omega t \, e^{-st} \, dt$$

$$= A\int_0^\infty \sin\omega t \cdot e^{-st} \, dt$$

~~using ILATE rule we get~~

$$= A\{\cancel{\sin\omega t\, e^{-st}} - \cos$$

$$\because \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$L\{f(t)\} = \frac{1}{2j}L\{e^{j\omega t} - e^{-j\omega t}\} = \frac{1}{2j}\left[\int_0^\infty (e^{j\omega t}\cdot e^{-st} - e^{-j\omega t}\cdot e^{-st})\,dt\right]$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{(j\omega - s)t} - e^{-(j\omega + s)t}\right]$$

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$$= \frac{1}{2j}\left[\frac{e^{(j\omega-s)t}}{(j\omega-s)} + \frac{e^{-(j\omega+s)t}}{(j\omega+s)}\right]_0^\infty$$

$$= \frac{1}{2j}\left[\frac{(0-1)}{(j\omega-s)} + \frac{(0-1)}{(j\omega+s)}\right]$$

$$= \frac{-1}{2j}\left[\frac{1}{(j\omega-s)} + \frac{1}{(j\omega+s)}\right]$$

$$= \frac{-1}{2j}\left[\frac{j\omega+\cancel{s}+j\omega-\cancel{s}}{-\omega^2-s^2}\right]$$

$$= \cancel{\frac{-1}{2j}}\left[\frac{\cancel{2j}\omega}{\cancel{-}(\omega^2+s^2)}\right] = \frac{\omega}{\omega^2+s^2}$$

$$= \frac{\omega}{s^2+\omega^2} \quad \text{Ans.}$$

Similarly,

V) $f(t) = t^n$

The the Laplace transform of t^n is given by

$$\boxed{L\{t^n\} = \frac{\lfloor n}{s^{n+1}}}$$

VI) Hyperbolic sine and cosine Functions

Given: $f(t) = \cosh \beta t$

$\because \cosh \beta t = \dfrac{e^{\beta t}+e^{-\beta t}}{2}$

and $\sinh \beta t = \dfrac{e^{\beta t}-e^{-\beta t}}{2}$

On putting the value of these two and solving it we get

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$$\boxed{L\{(\cosh \beta t)\} = \frac{s}{s^2-\beta^2}}$$

also $$\boxed{L\{\sinh \beta t\} = \frac{\beta}{s^2-\beta^2}}$$

VI.) Damped sine and cosine function

Damped sine function is given by:

$$f(t) = e^{-at}\sin \beta t$$

on taking the Laplace transform of it we get:

$$\boxed{L\{e^{-at}\sin \beta t\} = \frac{\beta}{(s+a)^2+\beta^2}}$$

Similarly

For damped cosine function

The value of Laplace transform is given by

$$\boxed{L\{e^{-at}\cos \beta t\} = \frac{s+a}{(s+a)^2+\beta^2}}$$